Math 216 - Exam 1 Solutions
Fall 2001 - Hartlaub.

1. Paired Data => Use Wilcoxon Signed Rank

(a)

Test Stat	-10	-8	-6	-4	-2	0	2	4	6	8	10
Prob	1/16	1/16	1/16	1/8	1/8	1/8	1/8	1/8	1/16	1/16	1/16

(b)

Test Stat	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Prob	1/16	1/16	1/16	1/8	1/8	1/8	1/8	1/8	1/16	1/16	1/16

(c) The results will NOT depend on which of the two statistics is used to conduct the test. The value of the test statistic will change, but the p-values will remain the same.

2. Matched Pairs => Use Wilcoxon Signed Rank Test.

(a) Let $z_i = \text{Control}_i - \text{Test}_i$ (y_i, x_i)

and θ = median of the population for the differences.

The point and interval estimators for θ are based on the Walsh averages of the differences.

$$\hat{\theta} = \text{median}\left\{ \frac{z_i + z_j}{2}, \ 1 \le i \le j \le 14 \right\} = 368$$

94.8% CI is (91, 1421) Stat > Nonpar > 1-Sample Wilcoxon.

(b) Reducing faults => $H_0: \theta = 0$ vs $H_1: \theta > 0$

(c) The C.I. in part (a) should not be used to test the hypotheses in part (b). C.I.'s can be used to [illegible] is needed to conduct an [illegible]

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3\. The A.R.E. is the ratio of sample sizes necessary to get the same power with two different tests. Since $\varepsilon(W, t) = 3$, this means that we would need approximately 3 times as many observations with the t-test to acheive the same power as the Wilcoxon rank sum test for exponential populations.

$$\left(\frac{n_t}{n_W} > 1 \Rightarrow n_t > n_W \Rightarrow \right.$$ Wilcoxon Rank sum test is better for exponential populations.

4\. Let p = proportion of artists that are left-handed.

(a) $H_0: p = .10$ vs $H_1: p > .10$

(b) Upper tailed test $\Rightarrow$ Find a lower C.B.

$$\hat{p} - z_\alpha \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

(7)

$$= \frac{18}{150} - 1.6449 \sqrt{\frac{\frac{18}{150}\left(\frac{132}{150}\right)}{150}} = .12 - .0436 = .0764$$

A 95% LCB for p is

$$(.0764, 1]$$

(3) Since $p_0 = .10$ is in the 95% LCB for p, the null hypothesis cannot be rejected. i.e. we do not have statistically significant evidence that artists are more likely to be left-handed.

(c) Test Stat: $B = 18$ $\quad B^* = \dfrac{18 - 150(.1)}{\sqrt{150(.1)(.9)}} = \dfrac{18-15}{3.6742} = .8165$

P-value: $P_0(B \geq 18) = 1 - P_0(B \leq 17)$ $\quad P_0(Z \geq .8165) \underset{MTB}{=} .2071$

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5. a. Assume: Distances are mutually indep & come from a population
3 that is continuous and symmetric about θ.

$H_0: \theta = 450$ vs. $H_1: \theta > 450$

8 Test Stat: $T^+ = 95.5$

p-value: .081

5 Conclusion: At any reasonable significance level (α = .05, .03, .02, or .01) we can't reject H_0 i.e., we do not have statistically significant evidence that the new ~~sign~~ design improves visibility.

b. $\hat{\theta} = \text{median}\left\{ \frac{z_i + z_j}{2},\ 1 \le i \le j \le 16 \right\}$ where z_i's are distances
$= 490$

6. a. $H_0: \eta_{OJ} = \eta_{ACID}$ vs $H_1: \eta_{OJ} > \eta_{ACID}$.

OR: $OJ = ACID + \Delta$ $H_0: \Delta = 0$ vs $H_1: \Delta > 0$

$W = 129$ p-value $= .0378$

At $\alpha = .05$ level, we reject H_0 and conclude that the OJ tends to produce larger ondontoblasts.

b. All $m \times n = 10 \times 10 = 100$ differences $\{ Y_j - X_i \}$ were computed and sorted. Then, the null distribution of the Wilcoxon Rank sum statistic was used to find the position constant (how many positions to count up from the bottom & down from the top.)

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5. (b) $1-\alpha = .936 \Rightarrow \alpha = .064 \Rightarrow \alpha/2 = .032$

$$C_{.064} = \frac{10(2(10)+10+1)}{2} + 1 - W_{.032}$$ ← Table A.6

$$= 155 + 1 - 130 = 26$$

93.6 % CI is $\left(\bar{U}^{(26)}, \bar{U}^{(100+1-26)}\right) \equiv \left(\bar{U}^{(26)}, \bar{U}^{(75)}\right)$

$\equiv (-.1, 9.1)$

(c) The null hypothesis in part (a) was rejected ~~[illegible]~~ because the one sided p-value is below the significance level of $\alpha = .06$. A 93.6% C.I. is equivalent to an $\alpha = 1 - .936 = .064$ level two-sided test. The duality between inferences for one sided tests ~~in~~ and those based on intervals only holds for confidence bounds.

(d.)